

Derivation of Adiabatic Lapse Rate

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Show that,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}$$

For an adiabatic process, $dU = -PdV$. Since $dU = C_v dT$ and $P = RT/V$

Then,

$$\frac{dT}{T} = -\frac{R}{C_v} \frac{dV}{V}$$

$$(1) \quad \begin{aligned} \frac{dT}{T} &= -(\gamma - 1) \frac{dV}{V} \\ \int_{T_1}^{T_2} \frac{dT}{T} &= \int_{V_1}^{V_2} -(\gamma - 1) \frac{dV}{V} \\ \ln\left(\frac{T_2}{T_1}\right) &= (\gamma - 1) \ln\left(\frac{V_1}{V_2}\right) \\ \frac{T_2}{T_1} &= \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} \end{aligned}$$

If

$$\left(\frac{P_2}{P_1}\right)^{(\gamma-1)} \left(\frac{T_1}{T_2}\right)^{(\gamma-1)} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = \frac{T_2}{T_1}$$

Then

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}$$

Similar to equation 1, we have

$$\frac{dT}{T} = \left(\frac{\gamma - 1}{\gamma}\right) \frac{dP}{P}$$

Since

$$\frac{dP}{P} = \frac{Mg}{RT} dz$$

Then

$$\frac{dT}{dz} = -\left(\frac{\gamma-1}{\gamma}\right) \left(\frac{Mg}{R}\right) \equiv \text{adiabatic lapse rate}$$