Derivation of Adiabatic Lapse Rate
http://climatehoax.ca/
Show that,

$$\boldsymbol{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}}$$

For an adiabatic process, dU = -PdV. Since dU = Cv dT and P = RT/V
Then,

$$\frac{dT}{T} = -\frac{R}{Cv}\frac{dV}{V}$$

(1)
$$\frac{dT}{T} = -(\gamma - 1)\frac{dV}{V}$$

$$\int_{T1}^{T2}\frac{dT}{T} = \int_{V1}^{V2} -(\gamma - 1)\frac{dV}{V}$$

$$ln\left(\frac{T_2}{T_1}\right) = (\gamma - 1)ln\left(\frac{V_1}{V_2}\right)$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)}$$

If

$$\left(\frac{P_2}{P_1}\right)^{(\gamma-1)}\left(\frac{T_1}{T_2}\right)^{(\gamma-1)} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = \frac{T_2}{T_1}$$

Then

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/2}$$

Similar to equation 1, we have

$$\frac{dT}{T} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{dP}{P}$$

Since

$$\frac{dP}{P} = \frac{Mg}{RT}dz$$

Then

$$\frac{dT}{dz} = -\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\mathrm{Mg}}{\mathrm{R}}\right) \equiv \text{adiabatic lapse rate}$$